(Solid State) Unit - III Band Theory

(Q) Draw the band diagram of n-type and p-type semiconductor?

(ii) Draw the band diagram of p-n diode

(iii) Define direct band gap and indirect

→ (i)

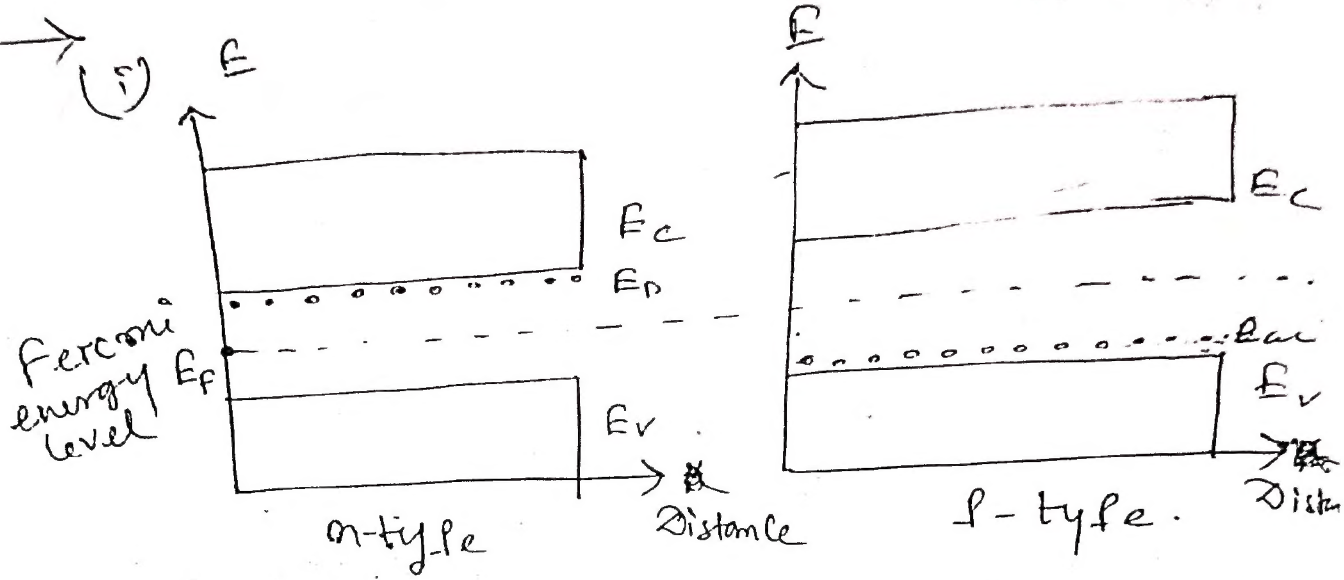
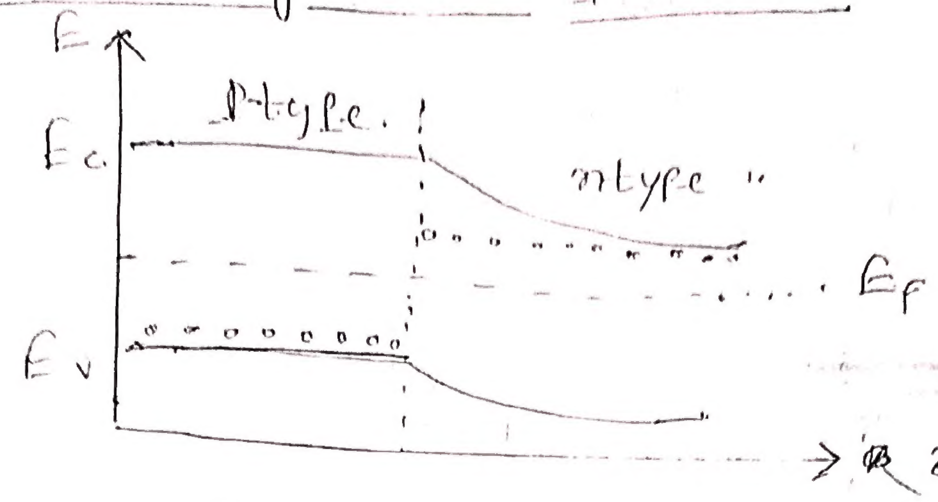


fig: energy diagram for p-type and n-type semi conductor.

In an n-type ~~extrinsic~~ semiconductor the electrons are near the conduction band and few holes exist near valence band, so it as expected that the fermi level moves closer to conduction band for an n-type semiconductor.

Similarly in p-type semiconductor with many holes near the valence band and some electron near the ~~valence~~ conduction band. so the fermilevel moves closer to valence band.

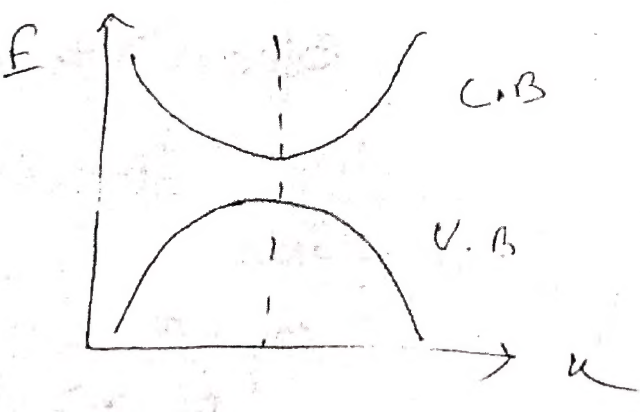
Band Diagram for p-n-diode:-

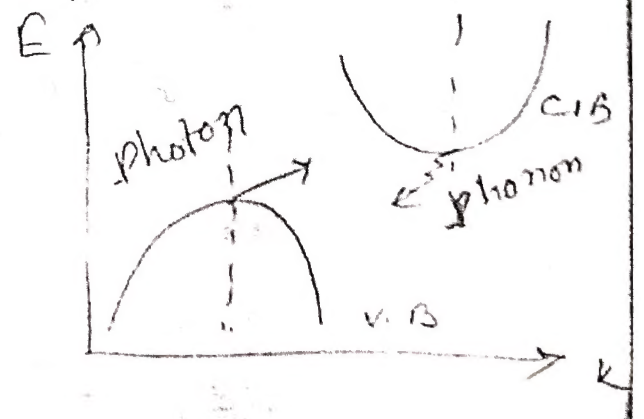


What do you mean by direct and indirect band gap:

The ~~minimal~~ energy value in conduction band and ~~maximal~~ energy value in valence band ~~energy~~ are characterized by a certain value of K (crystal momentum) in the brillouin zone.

If the K (crystal momentum) ~~value~~ vactor of electron and holes is same for C.B and V.B the band gap is called direct gap. and if the crystal momentum vector (K) is different the band gap is called indirect band gap.





(x) State Bloch Theorem :

According to free electron theory an electron moves in a constant potential V_0, the Schrodinger wave equ^n is given by,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi = 0 \longrightarrow (i)$$

The solution of above equ^n is given by,

$$\psi(x) = A e^{\pm ikx}$$

where, $K^2 = \frac{2m}{\hbar^2}(E - V_0)$

$$\Rightarrow (E - V_0) = \frac{K^2\hbar^2}{2m} = \frac{p^2}{2m} = E$$

In a crystal lattice an electron m... in a Periodic Potential $V(x)$.

The Schrodinger wave equ^n is given by

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\{E - V(x)\}\psi = 0 \longrightarrow (ii)$$

$V(x+a) = V(x)$ where a is lattice const.

This is known as Bloch theorem

The solution of equ^n (ii) is given by,

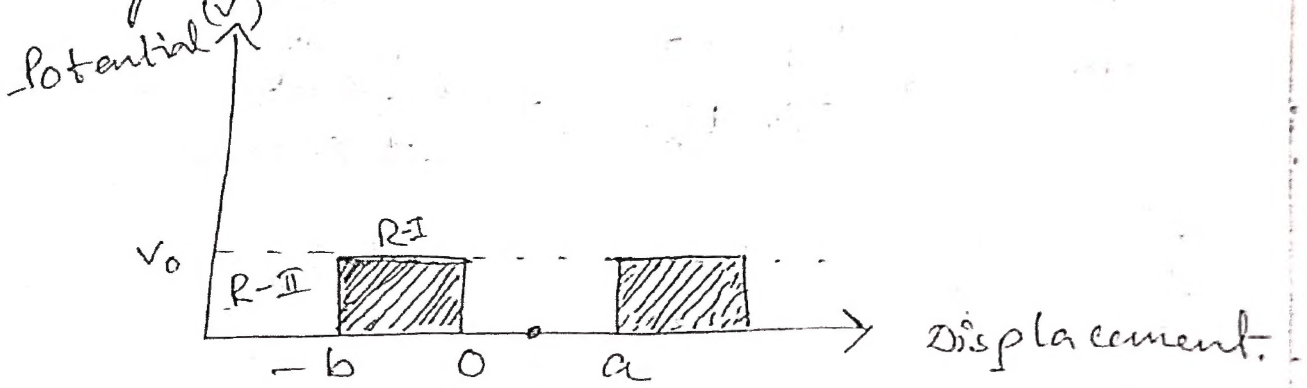
$$\psi_k(x) = U_k(x)\, e^{\pm ikx}$$

$U_k(x)$ = Periodicity of potential.

(x) (i) Prove Kronig - Penney model of Band Theory.
(ii) Draw Band Diagram from Kronig-Penney model.

→ (i) <u>Kronig - Penney Model:</u>

The potential of electron varries periodically with periodicity of ion-core (nuclues) and the potential of electron is zero near nucleus and maximum, when it is lying between adjacent nuclei which are separated by intere-atomic spacing 'a'.



Applying schrodinger time independent equ^n,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

For Region-II,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0 \quad \text{——— (i)}$$

For Region-I,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi = 0 \quad \text{——— (ii)}$$

$$\therefore \frac{d^2\psi}{dx^2} + \alpha^2\psi = 0 \quad \text{---(iii)}$$

$$\frac{2m}{\hbar^2}E = \alpha^2$$

$$\frac{2m}{\hbar^2}(E - V_0) = -\beta^2$$

$$\therefore \frac{d^2\psi}{dx^2} - \beta^2\psi = 0 \quad \text{---(iv)}$$

From Bloch theorem of the solⁿ of schrodinger equⁿ for a Periodic Potential.

$$\therefore \psi_K(x) = e^{ikx} U_K(x)$$

$U_K(x)$ Periodicity of the crystal lattice.

$$\frac{d\psi}{dx} = e^{ikx} U_K'(x) + (ik) U_K(x) e^{ikx}$$

$$\frac{d^2\psi}{dx^2} = (ik) e^{ikx} U_K'(x) + e^{ikx} U_K''(x) - K^2 U_K(x) e^{ikx} + (ik) e^{ikx} U_K'(x)$$

$$= e^{ikx} U_K''(x) + 2ik e^{ikx} U_K'(x) - K^2 U_K(x) e^{ikx} \quad \text{---(v)}$$

Using equⁿ (v) in equⁿ (iii) & (iv)

$$\frac{d^2 U_k(x)}{dx^2} + 2ik\frac{dU_k(x)}{dx} + (\alpha^2 - k^2)U_k(x) = 0 \quad \text{----- (vi)}$$

$$\frac{d^2 U_k(x)}{dx^2} + 2ik\frac{dU_k(x)}{dx} - (k^2+\beta^2)U_k(x) = 0 \longrightarrow \text{(vii)}$$

General solution of equ^n (vi) and (vii) is given by,

$$U_1 = A e^{i(\alpha-k)x} + B e^{-i(\alpha+k)x} \longrightarrow \text{(ix)}$$

$$U_2 = C e^{(\beta-ik)x} + D e^{-(\beta+ik)x} \longrightarrow \text{(x)}$$

Solution of equ^n (10) and (9) given by,

$$\boxed{\frac{P}{\alpha a}\sin\alpha a + \cos\alpha a = \cos ka}$$

This is known as Kronig Penney Model.

P = Power of Potential.

<u>Case - 1</u> : $P \longrightarrow \infty$

$$\frac{\sin\alpha a}{\alpha a} = 0$$

$\therefore \alpha a = n\pi$

$\therefore \alpha = \frac{n\pi}{a}$

$\therefore \alpha^2 = \frac{n^2\pi^2}{a^2} \Rightarrow \frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{a^2} \Rightarrow E = \frac{n^2\pi^2\hbar^2}{2ma^2}$

This case represents insulators.

Case : II :

$$P \to 0$$

$$\therefore \cos\alpha a = \cos Ka.$$

$$\alpha a = Ka$$

$$\therefore \alpha = K$$

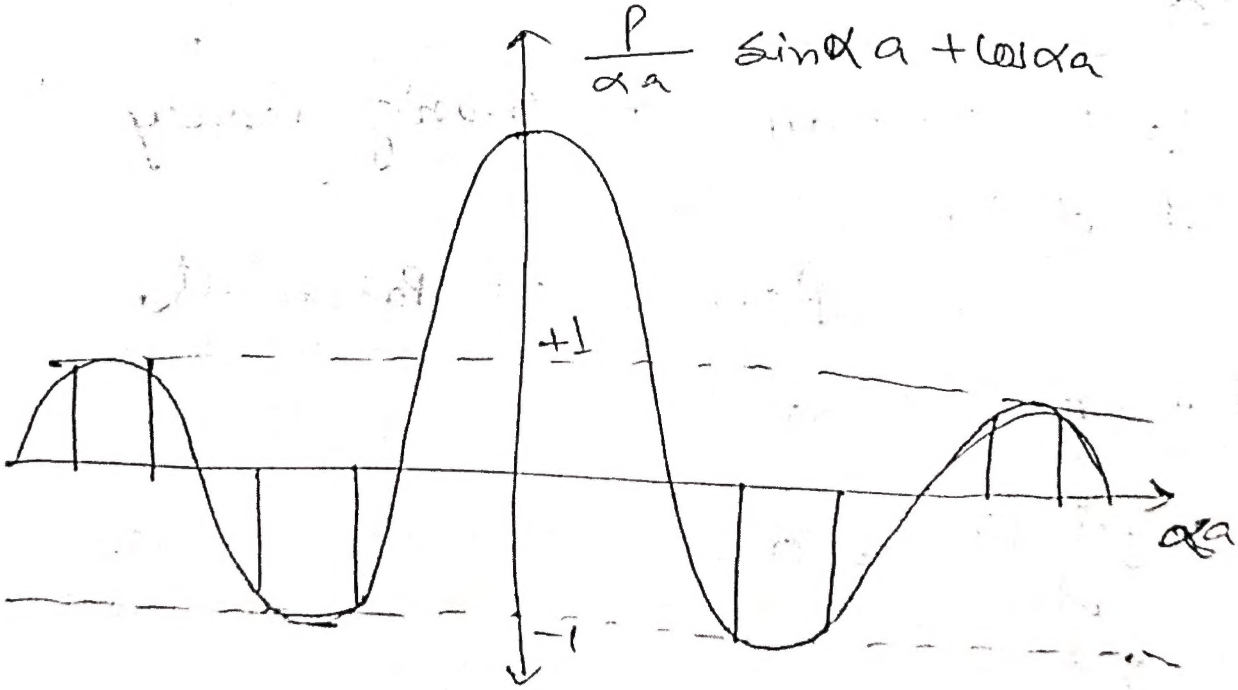
So, $\alpha^2 = K^2$

$$\frac{2mE}{\hbar^2} = K^2$$

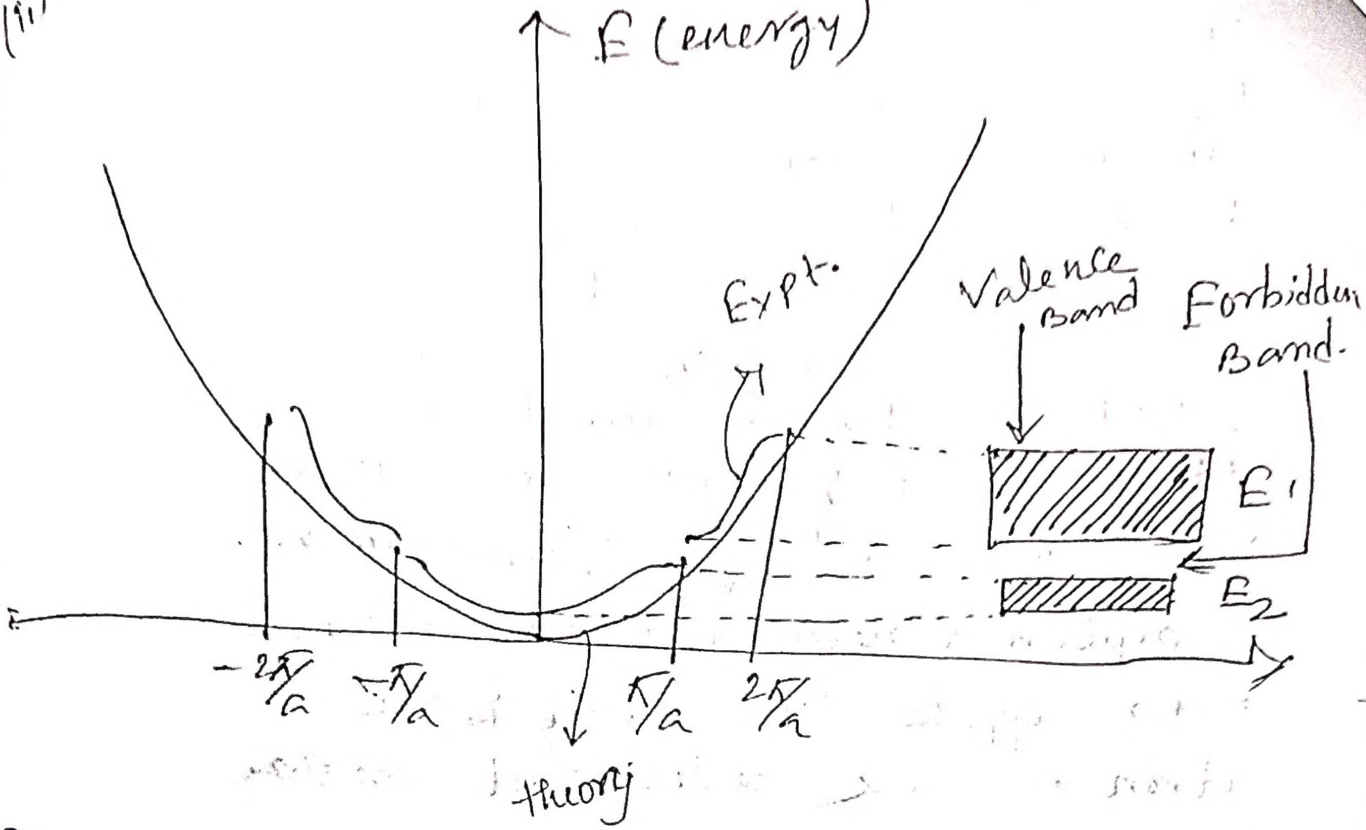
$$\therefore E = \frac{K^2\hbar^2}{2m} = \frac{p^2}{2m}$$

$\therefore$ fo metals.

Varriation of $\frac{P}{\alpha a}\sin\alpha a + \cos\alpha a$ Vs αa.



(ii) (*) E - K curve for free electron: $E = \frac{\hbar^2 k^2}{2m}$



(*) (i) What do you mean by "effective mass"?

(ii) Write down the significance of "effective mass"?

→ The energy of free electron, $E = \frac{\hbar^2 k^2}{2m}$

$\therefore m = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$

m is called effective mass of electron.

From ~~force~~ free electron theory the E-K graph represents a parabola which changes periodically with $\frac{d^2E}{dk^2}$ remains const.

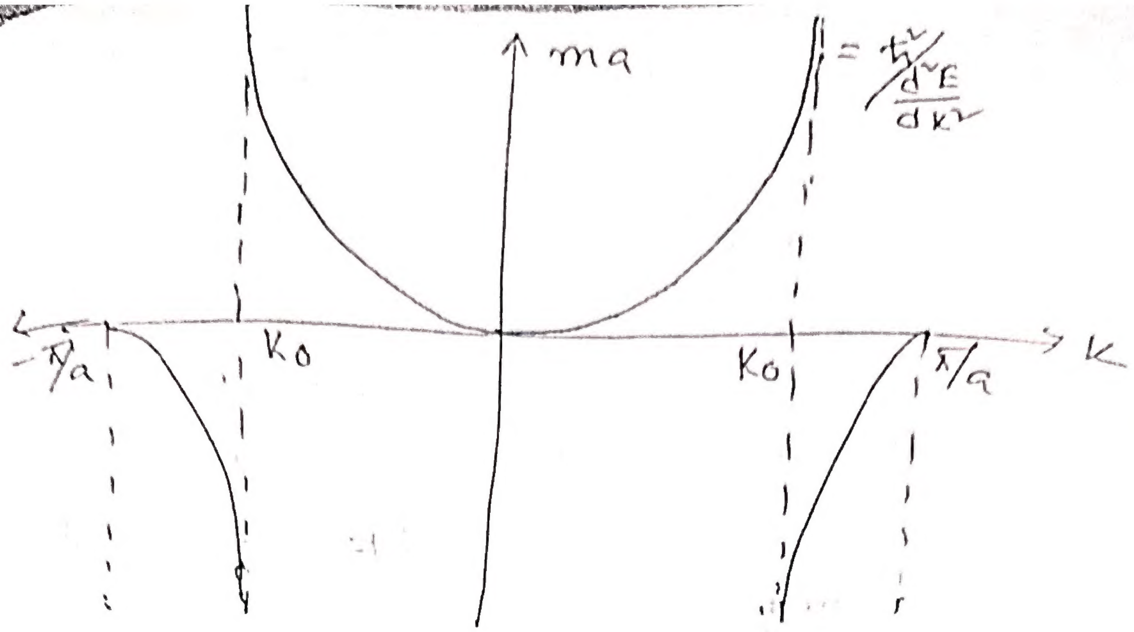
But which an electron moves through a periodic potential in a lattice crystal the relation between E-K is no longer true and curvature of E-K graph changes.

Significance of effective mass:-

The effective mass is a real new concept and arises because of the interaction of the electron wave packet with the periodic lattice. If there is strong binding force between the electron and the lattice, it will be difficult for the electrons to move, meaning thereby that the electron has acquired a large effective mass.

To explain a negative effective mass; Let ~~us~~ suppose that there is an electron, with k value just less than π/a at the boundary. It will manage to make through the crystal. But then suppose that a field is applied which should accelerate it and increase k. As the electron respond to the field it will meet the condition for Bragg reflection and will be scattered back in the opposite direction. In this way, it will behave like a particle with negative charge and negative mass.

It is clear that the effective mass is positive in the lower part of the band (lower value of k) and negative in upper band (higher value of k)



(i) Define Hall-effect. (ii) Discuss Hall-effect for a conductor?

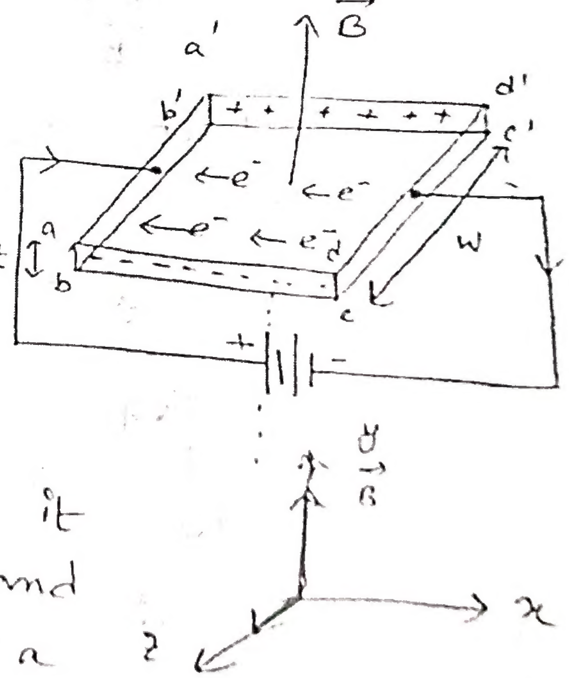
→ (i) It was suggested by Prof. E. H. hall, 1897. A magnetic field is applied perpendicular to the direction, of current in a conductor, then a transverse electric field is developed. This electric field is perpendicular to both the direction of current and magnetic field. This phenomenon of production of Electric field is known as "Hall effect". This field is called Hall field.

(ii) when a magnetic field applied, then e^-'s are feel some force, which is

$\vec{F} = -e(\vec{v}_d \times \vec{B})$,

V_d = drift velocity,

and will accumulate there it make's face abcd -ve and a'b'c'd' is +ve. as a result a potential diff^n or Hall voltage (V_H) is produced.



force produced 11

... direc^n.

Force due to Magnetic field,

$$F_B = e V_d B \sin 90°$$

$$= e V_d B \longrightarrow (1)$$

Magnetude of force due to electric field,

$$F_E = e E_H \longrightarrow (2)$$

at Equilibrium,

$$F_B = F_E$$

$$\Rightarrow e V_d B = e E_H$$

$$\Rightarrow V_d = \frac{E_H}{B} \longrightarrow (3)$$

the current in slab,

$$I = A n e V_d$$

$$I = w t n e V_d \qquad [\because A = wt]$$

$$V_d = \frac{I}{wtne} \longrightarrow (4)$$

using equ^n (3) and (4),

$$\frac{E_H}{B} = \frac{I}{wtne} \rightarrow (5)$$

$$\text{But, } E_H = \frac{V_H}{w} \longrightarrow (6)$$

from (5), $\Rightarrow$

$$\frac{V_H}{\cancel{w}B} = \frac{I}{\cancel{w}tne}$$

$$\therefore \boxed{V_H = \left(\frac{1}{ne}\right) \cdot \frac{BI}{t}}$$

this is the Expression of Hall voltage;

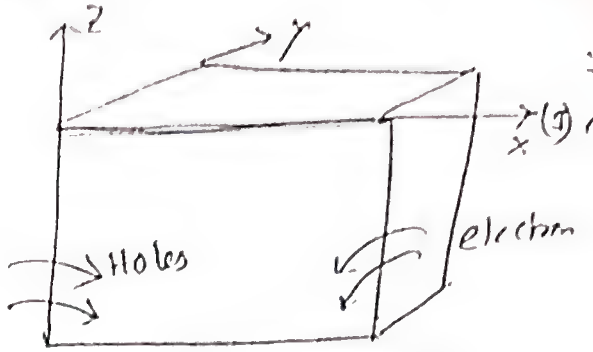
Hall co-efficient, $= \frac{1}{ne} = \frac{V_H t}{BI}$,

Hall resistance, $R_H = \frac{V_H}{I} = \frac{B}{net}$.

Hall co-efficient is +ve for, p-type semiconductor, and -ve for, n-type semiconductor.

(*) Discuss Hall effect for p-n diode semiconductor?

→ We consider a rectangular block of p-n semiconductor subjected to a electric field or current, along x-axis this will arise current density J_x in x-axis, electric field E_x in x-axis,



let a magnetic field B_z is applied along z axis, the effect of magnetic field applied a force on the charge carrier in -ve y-direction. Both holes and electrons are drifted to the front surface, Some of the holes and electrons recombined in the front surface. But there is a net surface charge on the front surface. The current density J_x due to electric field E_x.

$$J_x = \underbrace{n e \mu_e E_x}_{\text{for } e^-} + \underbrace{p e \mu_p E_x}_{\text{for holes}}$$

n: no. of e^- per unit volume

p = no. of hole's per unit volume

μ_e = mobility of e^-

μ_p = " " holes

Due to e^-, electric field is developed in -ve y-direction.

$E_y = -v_x B_z$

$$E_y = -\mu_e E_x B_z \longrightarrow (1) \qquad \mu_e = \frac{v_x}{E_x}$$

So, current density in y-direction due to electrons

$$J_{ye} = -n e \mu_e E_y$$

$$= n e \mu_e^2 E_x B_z \longrightarrow (ii) \quad [\text{using } (i)]$$

to holes,

$$J_{yh} = -pe\mu_p^2 E_x B_z \longrightarrow \text{(iii)}$$ [as charge is +ve]

So, current density in y-direcⁿ,

$$J_y = J_{ye} + J_{yh}$$

$$= eE_xB_z(n\mu_e^2 - p\mu_p^2)$$

At equilibrium the current density in x-direction = Current density in y-direction.

$$J_x = J_y$$

$$\Rightarrow ne\mu_eE_x + pe\mu_pE_x = eE_xB_z(n\mu_e^2 - p\mu_p^2)$$

$$\Rightarrow B_z = \frac{n\mu_e + p\mu_p}{n\mu_e^2 - p\mu_p^2}$$

Now, $$R_H = \frac{-\mu_e E_x B_z + \mu_p E_x B_z}{ne\mu_eE_x + pe\mu_pE_x}$$

$$\boxed{R_H = \frac{\mu_p - \mu_e}{ne\mu_e + pe\mu_p}}$$

(*) Find hall-co-efficient of p-type semiconductor with concentration of holes. 3×10^{26} m^{-3}, Hence find the mobility. If the conductivity is 10^{7} (Ωm^{-1}).

$\longrightarrow$ $$R_H = \cancel{\frac{\mu_p - \mu_e}{(}} = \frac{1}{nq} = \frac{1}{3\times10^{26}\times1.6\times10^{-19}}$$

$$= 0.208\times10^{-7}\ m^3/c$$

$$\mu = R_H\sigma = 0.208\times10^{-7}\times10^{7} = 0.208\ \Omega\cdot m^2/c$$

Super Conductivity

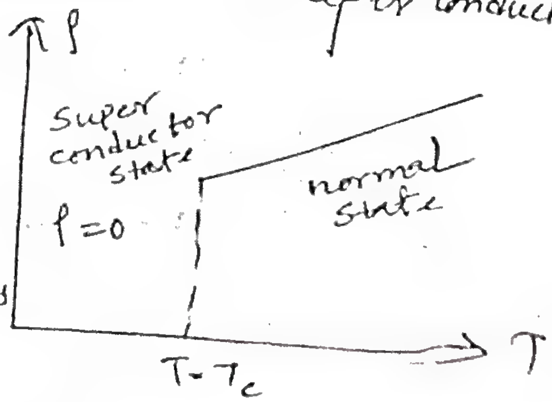
(i) Explain superconductivity with an Example.
(ii) Define Transition Temperature.

→ (i) We know that conductivity of metal increases with a decreasing in temperature. Theoritically resistivity of a metal, $\rho = \rho_0(1+\alpha T)$

α = Temperature co-efficient of Resistivity.
T = Temp (K).

fig :- Theoritical graph of ρ Vs T for Conductor.

But in Practical case for many conductors and alloy. It's observed that with decreasing temperature resistivity decreases and at a certain temperature the resistivity is become zero, that's conductivity infinity. This metals and alloys are known as super conductors and this phenomenon is known as super conductivity.



In 1911 1st time superconductivity was observed by scientist K. Onnes for mercury (Hg) at temp. 4.34K by using liquid He.

(ii) Transition temperature :- The temp. at which normal conductor and alloy are converted to super conductor. is known as transition temperature.

For Hg transition temp. 4.32 K.

(*) Write down a practical use of super conductor.

→ In MRI mechine superconductor are used to diagnose disease in a human body.

(*) Write down 2 - important property of super conductor?

→ (i) resistivity of a super conductor is zero

(ii) A superconductor is a perfect diamagnet with susceptibility $\chi = -1$

(*) State Meissner effect of super conductivi

→ The total magnetic field through a conducti

$$\vec{B}_{net} = \mu_0 (\vec{M} + \vec{H}_{applied}) \quad \text{---} (i)$$

; $\vec{M}$ = magnetization vector.

$\vec{H}_{applied}$ = applied magnetic field.

For a super-conductor the net magneticfield is zero, the magnetic field applied to the superconductor is repulse by the internal magnetic field or magnetization the superconductor.

from (i) $\vec{B}_{net} = 0$,

$$\Rightarrow \vec{M} + \vec{H}_{applied} = 0 \Rightarrow \frac{\vec{M}}{\vec{H}_{applied}} = -1$$

$\Rightarrow \chi = -1$

For a perfect diamagnet magnetic susceptibility χ is -1, that is superconductor behave as ~~perfect~~ diamagnet. This phenomenon is known as Meissner effect.

(*) Define critical magnetic field ~~at which~~ and ~~superconductor~~ critical current density?

→ The magnetic field at which a superconductor becomes a normal conductor is called critical magnetic field (H_c).

Critical magnetic field destroy's the superconductivity of the material. Critical magnetic field depends on temperature, the empirical or experimental relation between (H_c) and T is given by,

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right], \quad T < T_c$$

$$= 0 \quad ; \quad T > T_c$$

where
$H_c(0)$ = Critical magnetic field at 0 kelvin
T_c = Transition Temp.
Critical "

When current follows through a superconductor. This current is sufficient to produce critical magnetic field for the superconductor.

Conducting state will be destroy. This limit of current is known as critical current (I_c) and corresponding current density is called critical current density (J_c).

Critical current density depends on the temperature. The empirical relation between current density J_c and temperature T is given by

$$J_c(T) = J_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right], \quad T < T_c$$

$$= 0, \quad T > T_c$$

T_c = Critical Temp.

$J_c(0)$ = Critical current density at 0K.

T = Temp. in Kelvin.

For good superconductor T_c, H_c, J_c should be high.

Define Type-1 and Type-2 superconductor with Example.

→ The superconductors which follow meissner effect is called Type-1 superconductor.

Type-1 superconductor has one value of critical magnetic field.

Ex. of type-1 is :- Hg, Pb, Nb etc.

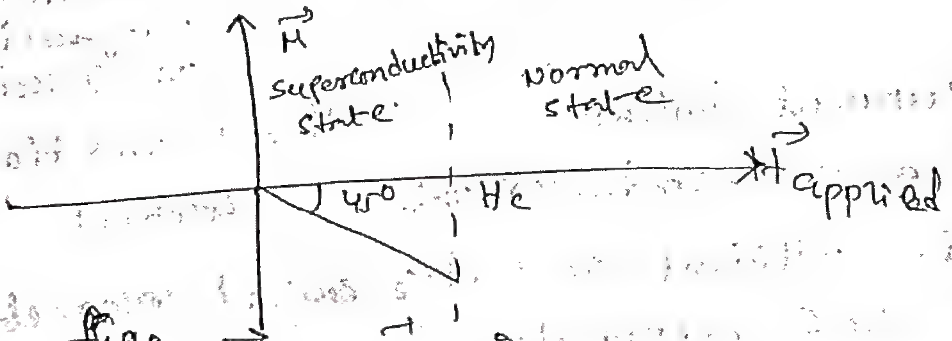


fig:- $\vec{M}$ Vs $\vec{H}$ applied.

The superconductor which does not follow meissner effect and has 2-critical magnetic field is known as Type-2 Super Conductor.

Ex — Nb_3Sn, Nb_3Al etc.

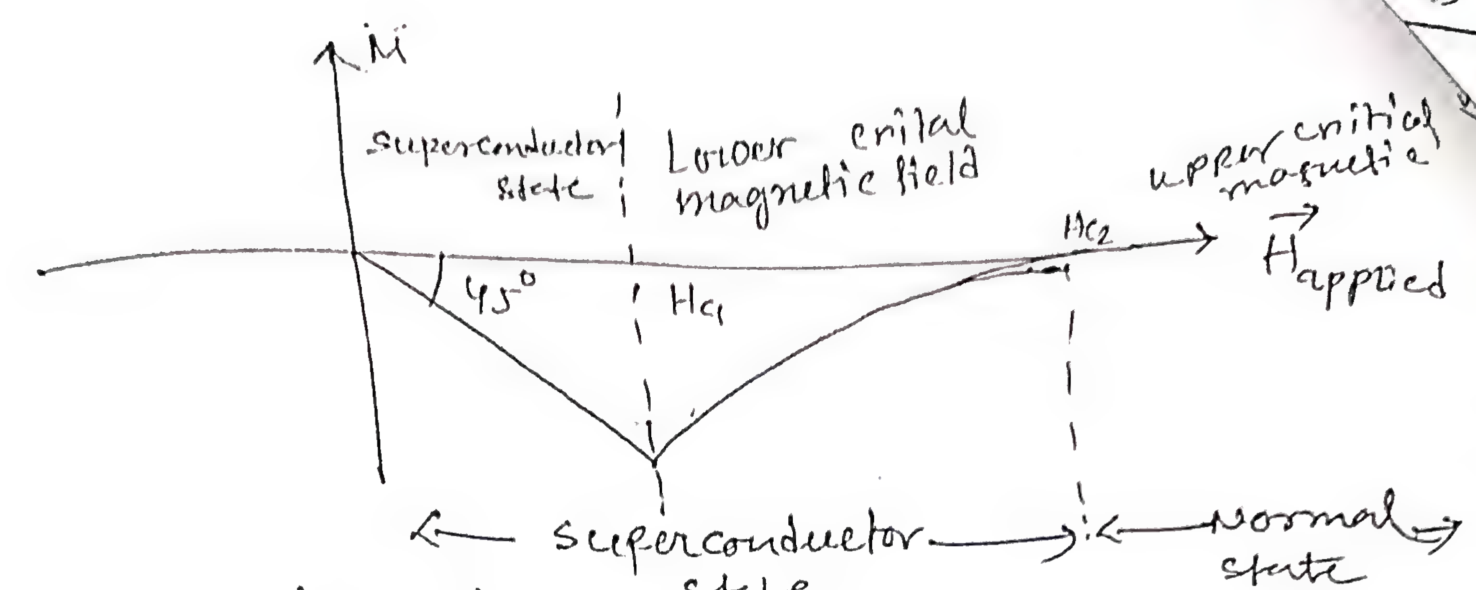


fig:- $\vec{M}$ vs $\vec{H}$applied for Type-2 Super Conducto

Upper critical magnetic field of Type-2 super-conductor is very high nearly 30-60 Tesla depending on superconductor.

(*) Critical magnetic field of a Type-1 Superconductor in '0' kelvin is 0.3 Tesla and critical temperature 15K. Find the critical magnetic field at T=5K.

→ $$H_c(T) = H_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

$$= 0.3\left[1-\left(\frac{\cancel{5}}{\cancel{15}\,3}\right)^2\right]$$

$$= 0.3 \times \frac{8}{9}$$

$$= 0.267\ T.$$

(i) Derive London's equⁿ of super conductivity?

(ii) Find an expression for London penetration Depth. Hence Define London Penetration Depth?

(iii) Write down the relation between London Penetration Depth and temp?

→ The electrodynamic equations of the super conductor was given by Heinz and Fritz London.

As discussed in Meissner effect the magnetic field inside the superconductor is not drop zero suddenly. The penetration of magnetic field through a super conductor is explained by using London's equation.

London's considered that they are two types of electron inside a super conductor,

→ Super Conducting electron
→ normal electron.

<u>1st London equⁿ:-</u>

We considered, n_s is number ~~of~~ density of super conducting electron.

v_s = velocity of super conducting electron.

force on super conducting electron due to electric field = $-eE$,

E = applied electric field.

e = charge.

$\therefore \; m\frac{dv_s}{dt} = -eE \longrightarrow (1)$.

Current density, $J = -n_s v_s e$,

or, $v_s = -\frac{J_s}{en_s}$, where,

v_s = velocity of super conducting electron,

$$\frac{dv_s}{dt} = -\frac{1}{en_s}\cdot\frac{dJ_s}{dt} \rightarrow (11)$$

J_s = super conducting current density.

Replacing equ^n (11) in (1) =>

$$-m\frac{1}{en_s}\cdot\frac{dJ_s}{dt} = -eE$$

$$\Rightarrow \frac{dJ_s}{dt} = \frac{e^2 n_s E}{m},$$ This is known as London ~~theory~~ first equ^n.

<u>2nd London equation:</u>

$$\frac{dJ_s}{dt} = \frac{e^2 n_s E}{m} \; \cdots\cdots \text{first London equ}^n \longrightarrow (1)$$

Taking curl on the both side of equ^n (1)

$$\vec{\nabla}\times\left(\frac{d\vec{J_s}}{dt}\right) = \frac{e^2 n_s}{m}\left(\vec{\nabla}\times\vec{E}\right) \longrightarrow (11).$$

from maxwell equ^n,

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}.$$

from (11),

$$\vec{\nabla}\times\left(\frac{d\vec{J_s}}{dt}\right) = -\frac{e^2 n_s}{m}\frac{\partial\vec{B}}{\partial t}$$

e = charge of e^-,

n_s = number of supere conducting e^- per unit volume.

m = mass of e^-.

$$\Rightarrow \frac{\partial}{\partial t}\{\nabla \times J_s\} = -\frac{e n_s}{m}\frac{\partial B}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial t}\{\vec{\nabla} \times \vec{J_s}\} = -\frac{\partial}{\partial t}\{\frac{e^2 n_s}{m}\vec{B}\} \longrightarrow (iii)$$

Integrating both side of equ^n (iii) w.r.to time (t).

$$\int \frac{\partial}{\partial t}(\vec{\nabla} \times \vec{J_s})\,dt = -\int \frac{\partial}{\partial t}\left(\frac{e^2 n_s}{m}\vec{B}\right)dt.$$

$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

$\vec{\nabla}$, Delta operator

$\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{J}) = \frac{\partial}{\partial t}(\vec{\nabla}) \times \vec{J} + \vec{\nabla} \times \frac{\partial \vec{J}}{\partial t} = \vec{\nabla} \times \frac{\partial \vec{J}}{\partial t}$

$$\boxed{\vec{\nabla} \times \vec{J_s} = -\frac{e^2 n_s}{m}\vec{B}}$$

This is known as London 2nd equ^n of super conductivity.

again $\vec{B} = \vec{\nabla} \times \vec{A}$

$$\vec{\nabla} \times \vec{J_s} = \vec{\nabla} \times \left(-\frac{e^2 n}{m}\vec{A}\right)$$

$$\Rightarrow \boxed{\vec{J_s} = -\frac{e^2 n_s}{m}\vec{A}}$$

From Ampere's low of magnetism;

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I$$

I_s = Super Conducting current $= \iint \vec{J_s}\cdot d\vec{s}$

using stokes theorem,

$$\Rightarrow \vec{\nabla} \times \vec{B} = \mu_0 J_s \text{ --- (i)}$$

J_s = Super Conducting current density

Taking curl on the both side of equ^n (i),

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \mu_0 (\vec{\nabla} \times \vec{J_s})$$

$$\Rightarrow \vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \vec{\nabla}\cdot(\vec{\nabla}\cdot\vec{B}) = \mu_0 \left\{\vec{\nabla} \times \left(-\frac{n_s e^2 \vec{A}}{m}\right)\right\}$$

$$\Rightarrow -\nabla^2\vec{B} = -\frac{\mu_0 n_s e^2}{m}(\vec{\nabla}\times\vec{A})$$ $\left\{ \begin{array}{l} \vec{\nabla}\cdot\vec{B} = 0 \\ \vec{\nabla}\times\vec{A} = \vec{B} \end{array} \right.$

$$\Rightarrow \boxed{\nabla^2\vec{B} = \frac{\mu_0 n_s e^2}{m}\vec{B}}$$

$\epsilon_0 \rightarrow F/m$

$\mu_0 \rightarrow H/m$

for one dimension,

$$\therefore \frac{\partial^2\vec{B}}{\partial x^2} = \frac{\mu_0 n_s e^2}{m}\vec{B};$$

$$\frac{m}{\mu_0 n_s e^2} = [L^2],$$

$$\cancel{\frac{m}{\mu_0 n_s e^2}} = M$$

$$[n_s] = [L^{-3}]$$

$$\Rightarrow c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

$$\Rightarrow \mu_0\epsilon_0 = \frac{1}{c^2}$$

$$C = \frac{Q^2}{QV}$$

$$[\epsilon_0] = \left[\frac{(IT)^2}{ML^2T^{-2}}\right]$$

$$\Rightarrow \mu_0\left[\frac{I^2T^2}{ML^3T^{-2}}\right] = \frac{1}{[LT^{-1}]^2}$$

$$\Rightarrow \mu_0 = \left[\frac{ML^3T^{-2}}{I^2T^2L^2T^{-2}}\right]$$

$$= \left[\frac{MLT^{-2}}{I^2}\right]$$

$$[e] = [IT]$$

$$\lambda_L^2 = \left(\frac{m}{\mu_0 n_s e^2}\right)$$

$\lambda_L = [L] =$ Penetration depth,

$$\Rightarrow \frac{\partial^2\vec{B}}{\partial x^2} = \frac{1}{\lambda_L^2}\vec{B}$$

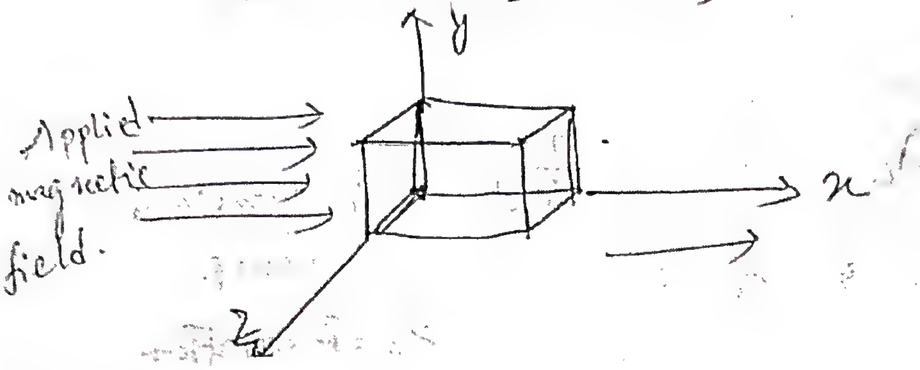
$$\Rightarrow \left(\frac{\partial^2\vec{B}}{\partial x^2} - \frac{1}{\lambda_L^2}\vec{B}\right) = 0$$

... non homogeneous differential equ^n.

$m^2 = \frac{1}{\lambda_L^2}$

$\Rightarrow m = \pm \frac{1}{\lambda_L}$

$\Rightarrow \vec{B} = c_1 e^{-\frac{1}{\lambda_L}x} + c_2 e^{\frac{1}{\lambda_L}x}$;



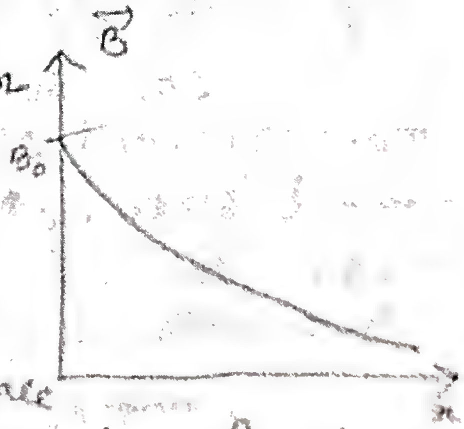
$\vec{B}$ = magnetic lines of force through Super Conductor.

The 2nd term indicate magnetic Induction increase with distance from the surface of superconductor which is not true, $\vec{B} = c_1 e^{-x/\lambda_L}$

Let, at, $x = 0$ (at the surface super conductor)

$\vec{B} = \vec{B_0}$, $c_1 = \vec{B_0}$ (magnetic at the surface)

$\vec{B} = \vec{B_0} e^{-x/\lambda_L}$



The distance at which the magnetic field is become $\frac{1}{e}$ times of magnetic field at the surface of superconductor is called London Penetration depth (λ_L).

The penetration depth is directly depends on temperature (T); with increase in temp. from transition Temp. (T_c) λ_L will increase.

The empirical relation between λ_L and T;

$$\lambda_L = \lambda_0 \left[1 - \frac{T}{T_c}\right]^{-1/2}$$; T_c = transition temp.

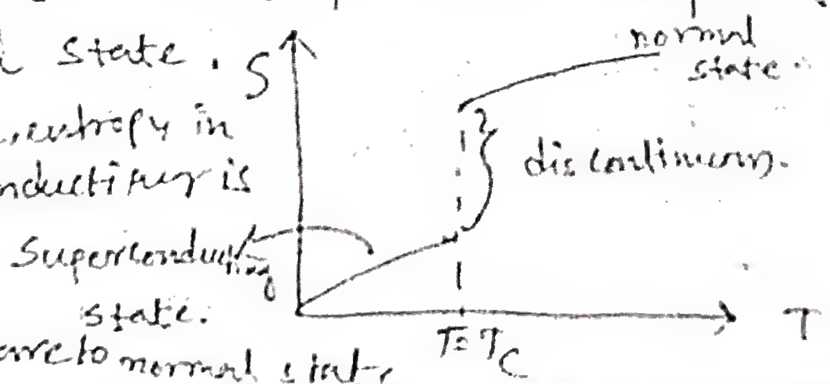
at $T = T_c \Rightarrow \lambda_L = 0$

λ_0 = Penetration depth at 0 kelvin.

(*) compare between entropy of super conducting state and normal state?

→ At super conducting state the resistance of super conductor is almost zero and in normal state due to electron - electron collision and electron - lattice point collision there is a significant resistance.

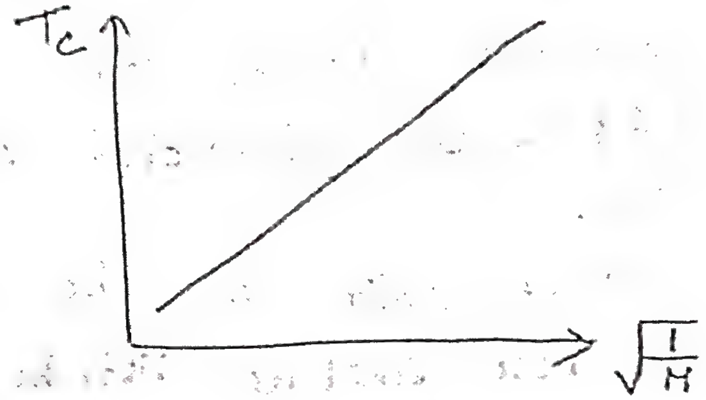
So, super conducting state is more ordered stystem as compare to normal state.



Therefore entropy in superconducting is less compare to normal state

(*) Discuss the variation of transition temp. with mass of the superconductor?

→ In 1950, E. Maxwell and Reynold, experimentally discover relation between transition temperature (T_c) and isotopic mass of superconductor by using isotopes of Hg.



M = mass of isotope.

$$\boxed{T_c \propto \sqrt{\frac{1}{M}}} \rightarrow \text{Experimental result.}$$

Hg^{200}, Hg^{202}, Hg^{201},

(*) Calculate the value of London penetration depth λ_0 at 0 K for Lead whose density is 11.3×10^3 kg/m³ and the atomic weight is 207.19 & T_c is 7.22 K.

→ $$\lambda_L^2 = \frac{m}{\mu_0 n_s e^2}$$

n_s = Superconducting density
$= 11.3 \times 10^3$ kg/m³

$M = 207.19$ amu
$= 207.19 \times 1.6 \times 10^{-27}$ kg

$$\lambda_L = \sqrt{\frac{207.19 \times 1.6 \times 10^{-27}}{4\pi \times 10^{-3} \times 11.3 \times 10^3 \times 1.6 \times 10^{-19}}}$$

* What is Cooper Pair in BCS Theory?

→ In solid state physics a Copper Pair or BCS Pair (Bardeen - Cooper - Schrieffer) is a losely bound pair of e^-'s with opposite spin's and moving with the same speed at low temp. This cooper pair is responsible for super conducting behaviour of material.

The concept of Cooper Pair is first described by American physicist Leon Cooper in 1956.

Cooper Pair is a boson particle and it obey bose - Einstine distribution.

* Write a short note on BCS theory.

(OR)

Explain qualitatively the main features of BCS theory?

→ (i) In a nutshell, BCS theory proposed that in superconducting state's at a very low temperature, two electrons form pair known as Cooper Pair. Thus all the e^-'s are Paired and formed large number of cooper pair

(ii) By over coming the mutual repulsion, the Pairing of -vely charged e^-'s becomes possible due to the exchange of a quantum of phonon between two electrons.

(iii) The quanta of phonons are formed due to lattice vibrations with Debye frequency. The energy of a quantum of phonon is, $\Delta = \hbar \omega_D$.

(iv) This energy $\Delta = \hbar \omega_D$ acts as the binding energy for the two electrons in the cooper pair.

(v) Since the lattice vibrates continuously with the continuous emission of quanta of phonons hence all the cooper pairs move as a single train through the lattice in coherence with the lattice vibration.

(vi) This quantum train of cooper pairs is described as Macroscopic quantum state with a single wave function:

$$\Psi = \Psi_0 e^{i\phi}$$

All the electrons have same phase ϕ.

(vii) The square of the amplitude of this wave funn is equal to the density of cooper pairs or super electron density n_s.

$$|\Psi_0|^2 = n_s$$

(viii) All the experimental observation in superconducting states namely lowering of entropy, Jumping of specific heat Isotope effect can be explained explicitly by BCS theory.

(8) Write down the characteristics of superconductivity.

→ the mean characteristics of superconductivity are —

(i) superconductor's has zero resistance/ infinite conductivity below the critical temp. / transition temp.

(ii) Super conductors repulse external magnetic field, behave as perfect diamagnetic this phenomenon is known as meissner effect.

(iii) Every superconductor has a specific critical temp. / transition temp. below which the conductor behave as super conductor and at other temp.'s behave as normal conductor.

(iv) Every super conductor has a critical magnetic field and critical current density. If the external magnetic field is greater than critical magnetic field. The super conductivity will be destroyed - similarly if the current density of super conductor is greater than critical current density, the super conductivity will be destroyed.

(v) the critical current density and critical magnetic field of superconductor changes with temp. below the transition temp.

or critical temp.

(vi) The External magnetic field applied to a superconductor is not instantly become zero, on the surface of superconductor, the external magnetic field decreases exponentially with distance from the surface of the super conductor, The distance or depth at which the external magnetic field is become $\frac{1}{e}$ ~~of~~ times of maximum magnetic field inside the superconductor is known as London penetration depth / London Penetration length.

(1) Difference between molecular bond and metallic bond?

→ Molecular

(i) It is weaker than metallic bond.

(ii) They have low melting point and boiling point.

Metalic

(i) It is stronger than molecular bond.

(ii) The have high ~~comparable to~~ melting point and boiling point compared to molecular bond

(2) → First para – Unit – I

(*) The Brillouin Zone is defined as the Wigner-Seitz cell of the reciprocal lattice

Primitive ; reciprocal